Odrediti opšte rješenje homogenog sistema

$$
\begin{aligned}
2x+y+z&=0\\
4x+2y+z&=0\\
6x+3y+z&=0\\
8x+4y+z&=0.
\end{aligned}
$$

Rj: Sistem riješimo Kroneker-Kapelijevom metodom

$$
\bar{A}=[A \mid b]=\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0\\ 4 & 2 & 1 & 0\\ 6 & 3 & 1 & 0\\ 8 & 4 & 1 & 0\end{array}\right]
\underset{\substack{III_v - I_v\cdot 3\\ IV_v - I_v \cdot 4}}{\overset{II_v - I_v\cdot 2}{\sim}}
\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & -2 & 0\\ 0 & 0 & -3 & 0\end{array}\right] \sim
$$

$$
\underset{IV_v + II_v\cdot(-3)}{\overset{III_v + II_v\cdot(-2)}{\sim}}
\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right]
\Rightarrow \quad \operatorname{rang} A = \operatorname{rang} \bar{A} = 2 < \underset{\text{broj nepoznati}}{3}
$$

pa prema Kroneker-Kapelijevoj metodi sistem ima beskonačno mnogo rješenja i jednu promjenjivu uzimamo proizvoljno

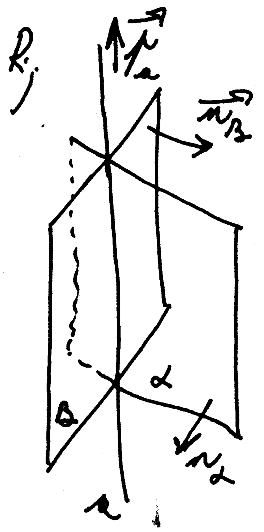
$$
\begin{aligned}
2x+y+z&=0\\
-z&=0\\
\hline
2x+y&=0\\
y&=-2x
\end{aligned}
$$

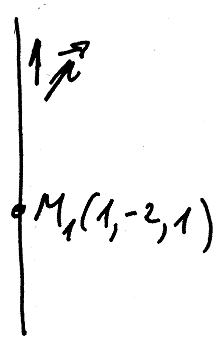
Ako je $x=t$ tada je $(t,-2t,0),\ t\in\mathbb{R}$ opšte rješenje sistema.

Ako uzmemo $y=t$ tada je $x=-\frac{1}{2}t$ pa je $\left(-\frac{1}{2}t, t, 0\right),\ t\in\mathbb{R}$ opšte rješenje sistema

Kroz tačku $M_1(1,-2,1)$ povući pravu paralelnu pravoj $\begin{cases} x-y+z-4=0 \\ 2x+y-2z+5=0 \end{cases}$.

R:





$$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$$ jednačina prave kroz tačku $M(x_1,y_1,z_1)$

$\alpha: x-y+z-4=0$

$\vec{n}_\alpha=(1,-1,1)$ vektor normale na ravan α

$\beta: 2x+y-2z+5=0$

$\vec{n}_\beta=(2,1,-2)$ vektor normale na ravan β

$\vec{p}_a \parallel \vec{p}$

$$\left.\begin{array}{l} \vec{p}_a \perp \vec{n}_\alpha \\ \vec{p}_a \perp \vec{n}_\beta \end{array}\right\} \Rightarrow \left.\begin{array}{l} \vec{p}_a \parallel \vec{n}_\alpha \times \vec{n}_\beta \\ \vec{p} \parallel \vec{p}_a \end{array}\right\} \Rightarrow \vec{p} \parallel \vec{n}_\alpha \times \vec{n}_\beta$$

$$\vec{n}_\alpha \times \vec{n}_\beta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & 1 & -2 \end{vmatrix} = (2-1)\vec{i}-(-2-2)\vec{j}+(1+2)\vec{k}=(1,4,3)$$

Za vektor pravca tražene prave mogu uzeti

$$\vec{p}=(1,4,3)$$

$$M_1(\overset{x_1}{1},\overset{y_1}{-2},\overset{z_1}{1})$$

$$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-1}{3}$$

jednačina tražene prave

(#) Vektor $\vec{a}=(1,0,1)$ izraziti kao linearnu kombinaciju vektora $\vec{b}=(2,1,0)$, $\vec{c}=(2,-1,1)$ i $\vec{d}=(0,2,0)$.

Rj. Želimo pronaći konstante β, γ i δ takve da

$$\vec{a}=\beta\vec{b}+\gamma\vec{c}+\delta\vec{d}$$

tj.

$$(1,0,1)=\beta(2,1,0)+\gamma(2,-1,1)+\delta(0,2,0)$$

$$\begin{aligned} 2\beta+2\gamma+0\delta&=1\\ \beta-\gamma+2\delta&=0\\ 0\beta+\gamma+0\delta&=1 \quad \Rightarrow \quad \gamma=1 \end{aligned}$$

$$2\beta+2=1 \quad \Rightarrow \quad 2\beta=-1$$

$$\beta=-\frac{1}{2}$$

$$\beta-\gamma+2\delta=0 \quad \Rightarrow \quad -\frac{1}{2}-1+2\delta=0$$

$$2\delta=\frac{3}{2}$$

$$\delta=\frac{3}{4}$$

Prema tome $\vec{a}=-\frac{1}{2}\vec{b}+\vec{c}+\frac{3}{4}\vec{d}$

(vektor $\vec{a}$ izražen kao linearna kombinacija vektora $\vec{b}$, $\vec{c}$ i $\vec{d}$).

(#) Izračunati: $\lim_{x\to 4}\left(\frac{x}{4}\right)^{\frac{1}{x-4}}$.

Rj. $\lim_{x\to 4}\left(\frac{x}{4}\right)^{\frac{1}{x-4}} \left(=1^{\frac{1}{0}}=1^{\infty}\right)$

neodređen izraz

Znamo da je $u^v = e^{\ln u^v}$. Zašto? Isto tako znamo da je $\ln u^v = v\ln u$. Prema tome

$$\lim_{x\to 4}\left(\frac{x}{4}\right)^{\frac{1}{x-4}} = \lim_{x\to 4} e^{\ln\left(\frac{x}{4}\right)^{\frac{1}{x-4}}} = \lim_{x\to 4} e^{\frac{1}{x-4}\ln\left(\frac{x}{4}\right)} =$$

$$= e^{\lim_{x\to 4}\frac{1}{x-4}\ln\left(\frac{x}{4}\right)} = e^{\lim_{x\to 4}\frac{\ln\frac{x}{4}}{x-4}} \left(= e^{\frac{0}{0}}\right) \underset{=}{\overset{L.P.}{=}}$$

$$= e^{\lim_{x\to 4}\frac{\left(\ln\frac{x}{4}\right)'}{(x-4)'}} = e^{\lim_{x\to 4}\frac{\frac{1}{\frac{x}{4}}\cdot\frac{1}{4}}{1}} = e^{\lim_{x\to 4}\frac{1}{x}} = e^{\frac{1}{4}} = \sqrt[4]{e}$$

(#) Ispitati i nacrtati f-ju $y=\dfrac{x+1}{(x-1)^2}$.

Rj. DEFINICIONO PODRUČJE

$(x-1)^2 \neq 0$

$x-1\neq 0$

$x\neq 1$

$\mathcal{D}:\ x\in\mathbb{R}\setminus\{1\}$

$x\in(-\infty,1)\cup(1,+\infty)$

PARNOST (NEPARNOST), PERIODIČNOST

Definiciono područje nije simetrično pa f-ja nije ni parna ni neparna.

F-ja nije periodična.

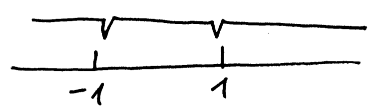
ZNAK, NULE, PRESJEK SA Y-osom

$f(0)=\dfrac{0+1}{(0-1)^2}=\dfrac{1}{1}=1$

$(0,1)$ je presjek sa y-osom

$y=0 \iff x+1=0 \iff x=-1$

$(-1,0)$ je nula f-je



x	$(-\infty,-1)$	$(-1,1)$	$(1,+\infty)$
x+1	−	+	+
Y	−	+	+

znak f-je

PONAŠANJE NA KRAJEVIMA INTERVALA DEFINISANOSTI I ASIMPTOTE

Za $x=1$ f-ja ima prekid.

$\lim\limits_{x\to 1-0} f(x)=\lim\limits_{x\to 1-0}\dfrac{x+1}{(x-1)^2}=\dfrac{1-0+1}{(1-0-1)^2}=\dfrac{2-0}{+0}=+\infty$

$\lim\limits_{x\to 1+0} f(x)=\lim\limits_{x\to 1+0}\dfrac{x+1}{(x-1)^2}=\dfrac{1+0+1}{(1+0-1)^2}=\dfrac{2+0}{+0}=+\infty$

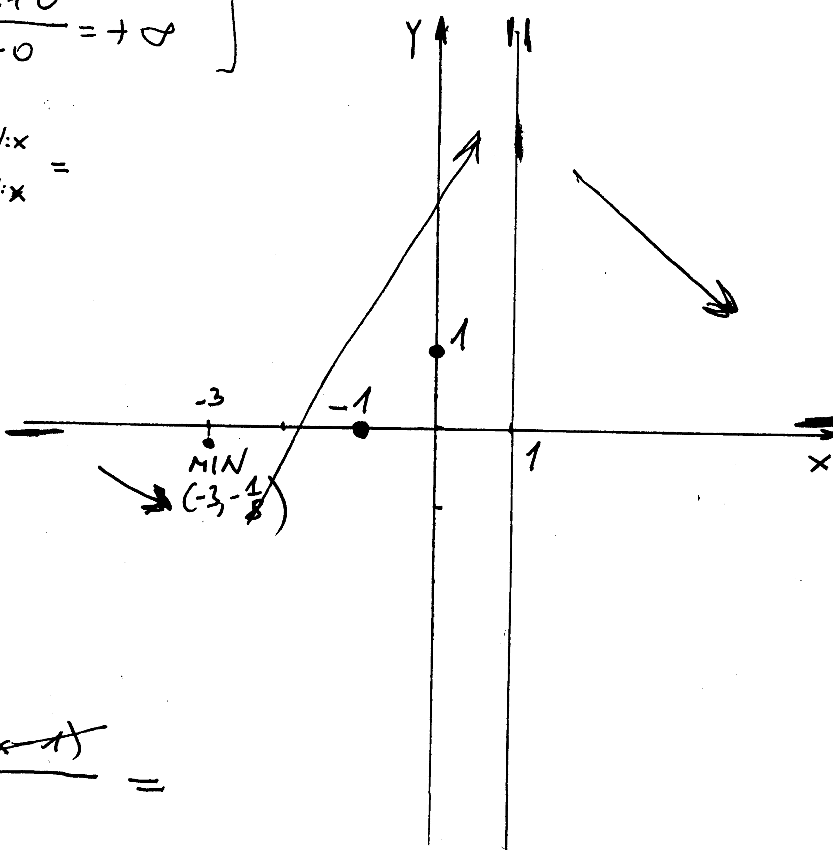
$\Rightarrow$ $y=1$ je V_oA_o

horizontalna asimptota

$\lim\limits_{x\to\pm\infty} f(x)=\lim\limits_{x\to\pm\infty}\dfrac{x+1}{(x-1)^2}=\lim\limits_{x\to\pm\infty}\dfrac{x+1}{x^2-2x+1}\,\begin{matrix}/:x\\ /:x\end{matrix}=$

$=\lim\limits_{x\to\pm\infty}\dfrac{1+\frac{1}{x}\to 0}{x-2+\frac{1}{x}\to 0}=0$

$\Rightarrow$ $x=0$ je H_oA_o



Poslije ovog koraka počinjemo skicirati graf f-je.

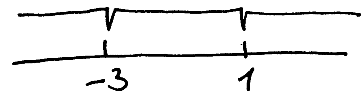
RAST I OPADANJE

$y'=\left(\dfrac{x+1}{(x-1)^2}\right)'=\dfrac{1\cdot(x-1)^{\cancel{2}\,1}-(x+1)2\cancel{(x-1)}}{(x-1)^{\cancel{4}\,3}}=$

$$= \frac{x-1-2x-2}{(x-1)^3} = \frac{-x-3}{(x-1)^3}$$

$$y' = (-1)\frac{x+3}{(x-1)^3}$$

$y' = 0$ akko $x = -3$



← prekidi y + nule y'

x	$(-\infty, -3)$	$(-3, 1)$	$(1, +\infty)$
y'	$-$	$+$	$-$
y	↘	↗	↘

MIN

tabela rasta i opadanja

$$f(-3) = \frac{-3+1}{(-3-1)^2} = \frac{-2}{(-4)^2} = -\frac{1}{8}$$

EKSTREMI F-JE

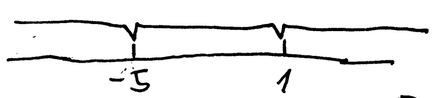
Na osnovu tabele rasta i opadanja vidimo da f-ja za $x=-3$ ima ekstrem u tački $(-3, -\frac{1}{8})$.

PREVOJNE TAČKE I INTERVALI KONVEKSNOSTI I KONKAVNOSTI

$$y'' = \left((-1)\frac{x+3}{(x-1)^3}\right)' = (-1)\frac{1\cdot(x-1)^{\cancel{3}} - (x+3)\,3\cancel{(x-1)^2}}{(x-1)^{\cancel{6}\,4}} = (-1)\frac{x-1-3x-9}{(x-1)^4}$$

$$= (-1)\frac{-2x-10}{(x-1)^4} = 2\cdot\frac{x+5}{(x-1)^4}$$

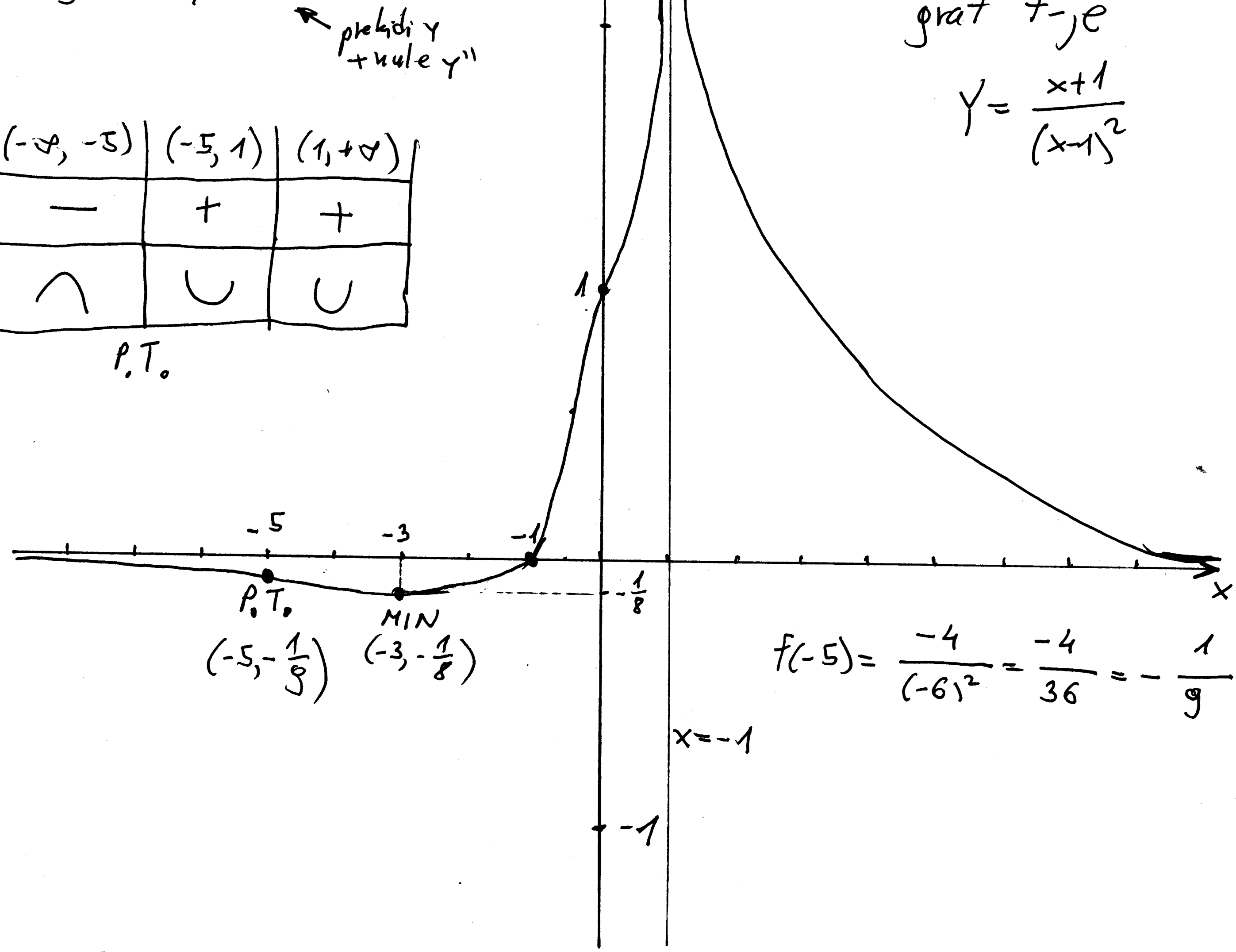
$y'' = 0$ akko $x = -5$



← prekidi y + nule y''

x	$(-\infty, -5)$	$(-5, 1)$	$(1, +\infty)$
y''	$-$	$+$	$+$
y	$\cap$	$\cup$	$\cup$

P.T.



$$f(-5) = \frac{-4}{(-6)^2} = \frac{-4}{36} = -\frac{1}{9}$$